

# Multi-robot SLAM via Distributed Extended Kalman Filters

Toshiki Sasaoka<sup>1</sup>, Yosuke Kishimoto<sup>1</sup> and Kiyotsugu Takaba<sup>1\*</sup>

<sup>1</sup>Department of Electrical and Electronic Engineering, Ritsumeikan University,  
 Shiga 525-8577, Japan (ktakaba@fc.ritsumei.ac.jp) \* Corresponding author

**Abstract:** This paper is concerned with the Simultaneous Localization and Mapping (SLAM) problem with multiple mobile robots. To reduce the computational and hardware costs, the distributed extended Kalman filter is applied to the multi-robot SLAM system. A numerical experiment demonstrates the effectiveness of the proposed method.

**Keywords:** multi-robot SLAM, distributed Kalman filter

## 1. INTRODUCTION

For practical applications of autonomous robots, it is essential for the robots to recognize their surroundings as well as their own positions. The problem of simultaneous estimation of self-location of a robot and its surrounding map is called the SLAM (Simultaneous Localization And Mapping). Recently, several attempts to improve the performance of the SLAM using multiple robots have been reported [1], [2]. The multi-robot SLAM methods can be classified into two types. In the first type, a base station or a host robot aggregates the information on the measurements from all the robots to compute the optimal estimates. In the second type, the individual robots carry out their estimation in a distributed and cooperative manner through local information exchanges without a base station.

In this paper, we consider the latter type of the multi-robot SLAM to reduce the computational and hardware costs. For this purpose, we adapt the distributed Kalman filter for linear systems [3], [4] to nonlinear systems, and apply the distributed extended Kalman filter to the multi-robot SLAM problem. We will verify the effectiveness of the proposed method by a numerical experiment using real data.

## 2. PROBLEM FORMULATION

Consider the multi-robot system consisting of  $n$  mobile robots and  $m$  landmarks (Fig 1). Each robot is equipped with a range sensor which measures the relative distances and angles to the other robots and landmarks. Let the dynamics of the  $i$ -th robot be expressed by the discrete-time state equation

$$x_{Ri,k+1} = f_R(x_{Ri,k}, u_{Ri,k}), \quad i = 1, \dots, n,$$

where  $x_{Ri,k}$  is the state of the robot  $i$  composed of the  $x, y$ -positions and the heading angle. The input  $u_{R,k}$  consists of the translational speed and the rate of change of the heading angle. Also, the  $x, y$ -position of the  $i$ -th landmark can be represented as

$$p_{Li,k+1} = p_{Li,k}, \quad i = 1, \dots, m.$$

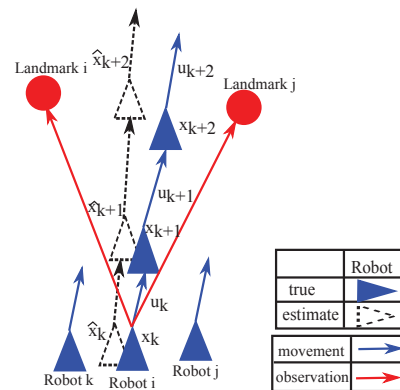


Fig. 1 Multi-robot SLAM

By putting these state equations together, we get the large-scale state equation

$$x_{k+1} = f(x_k, u_k) + w_k \quad (1)$$

$$x_k = \text{col}(x_{R1,k}, \dots, x_{Rn,k}, p_{L1,k}, \dots, p_{Lm,j}),$$

$$u_k = \text{col}(u_{R1,k}, \dots, u_{Rn,k}).$$

and  $w_k$  is the zero mean white Gaussian noise with variance  $Q_k$ . The measurement equation of the individual robots is given by

$$z_{i,k} = h_i(x_k) + v_{i,k}, \quad i = 1, \dots, n, \quad (2)$$

where the nonlinear map  $h_i$  characterizes the range sensor measurements, and  $v_{i,k}$  is the zero mean white Gaussian noise with covariance  $R_i > 0$ .

Each robot obtains the information of adjacent robots through its sensors. In other words, if the robot  $i$  detects the robot  $j$  from the sensor measurement, then the robot  $j$  is viewed as a neighbor of the robot  $i$ , and the robot  $i$  can obtain some information on the measurement and/or position of the robot  $j$ .

The multi-robot SLAM problem to be considered in this paper is stated below.

*Under the local communication constraints between the mobile robots, find a distributed and cooperative algorithm for estimating the positions of all the robots and landmarks based on the sensor measurements of the robots.*

Table 1 RMS Errors of Estimated Robot and Landmark Positions (DEKF) [mm], [°]

RMS Errors	Landmark1XY	Landmark2XY	Landmark3XY	Robot1XY	Robot1	Robot2XY	Robot2	Robot3XY	Robot3
Robot1	74.1194	68.5877	100.5274	33.8367	0.5264	25.4595	1.8522	83.9131	0.4891
Robot2	118.7813	86.0795	167.5418	40.3370	0.8541	29.3260	1.4431	54.8002	2.0840
Robot3	164.5549	100.9139	183.8171	58.3205	0.4636	38.4068	1.4783	48.7184	1.4943

Table 2 RMS Errors of Estimated Robot and Landmark Positions (LEKF) [mm], [°]

RMS Errors	Landmark1XY	Landmark2XY	Landmark3XY	Robot1XY	Robot1	Robot2XY	Robot2	Robot3XY	Robot3
Robot1	225.6740	206.2046	191.4172	18.0424	1.4886	14.2840	0.0970	-	-
Robot2	265.5475	229.4027	174.8148	34.9569	0.2432	22.0832	2.4300	38.7899	0.3633
Robot3	227.2787	192.1253	111.0759	-	-	17.2199	0.0662	21.2405	0.7909

### 3. DISTRIBUTED KALMAN FILTER

We adapt the distributed Kalman filter due to Saber [3], [4] to nonlinear systems, and apply the obtained algorithm to the multi-robot SLAM.

Assume that the a priori state estimate  $\bar{x}_j$  can be transmitted to the adjacent robots as well as the measurements  $z_j$ . The distributed Kalman filter contains the additional consensus term of  $\bar{x}_i$ 's in the measurement update step.

#### Distributed Extended Kalman Filter (DEKF)

##### Initialization:

$$\bar{x}_{i,0} = \zeta_0, P_{i,0} = \Pi_0, \epsilon > 0$$

##### Measurement Update ( $\hat{x}_{i,k}$ : a posteriori estimate )

$$\begin{aligned} \hat{x}_{i,k} &= \bar{x}_{i,k} + M_{i,k} \left( y_{i,k} - \sum_{j \in J_i} H_{j,k}^i \top R_j^{-1} h_j(\bar{x}_{i,k}) \right) \\ &\quad + \epsilon M_{i,k} \sum_{j \in N_i} (\bar{x}_{j,k} - \bar{x}_{i,k}) \\ M_{i,k} &= (P_{i,k}^{-1} + S_{i,k})^{-1} \\ y_{i,k} &= \sum_{j \in J_i} H_{j,k}^i \top R_j^{-1} z_{j,k}, \quad S_{i,k} = \sum_{j \in J_i} H_{j,k}^i \top R_j^{-1} H_{j,k}^i \end{aligned}$$

$N_i$  denotes the neighbors to the robot  $i$ , and  $J_i = N_i \cup \{i\}$ .

##### Time Update ( $\bar{x}_{i,k}$ : a priori estimate )

$$\begin{aligned} \bar{x}_{i,k+1} &= f(\hat{x}_{i,k}, u_k) \\ P_{i,k+1} &= F_{i,k} M_{i,k} F_{i,k}^\top + Q_k \\ F_{i,k} &= \left. \frac{\partial f(x, u)}{\partial x} \right|_{(x, u) = (\hat{x}_{i,k}, u_k)}, \quad H_{j,k}^i = \left. \frac{\partial h_j(x)}{\partial x} \right|_{x = \bar{x}_{i,k}} \end{aligned}$$

It may be noted that the communication cost for exchanging the state estimates in the above algorithm is negligible, if the robots are equipped with high speed Wi-Fi communication devices.

### 4. NUMERICAL EXPERIMENT

We collect the measurement data using 3 mobile robots and 3 landmarks under the same experimental setup as in [2] (Fig. 2). Because of the limited computational power of the robots, we carry out the DEKF algorithm off-line over MATLAB on a Windows 7 PC with Intel Core i5 3.2GHz CPU and 8GB memory. Because of the relative location of the robots in Fig. 2, the robot 1 cannot observe the robot 3, and the converse applies to the robot 3.

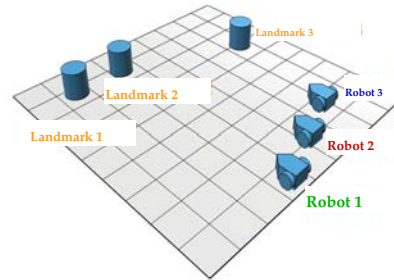


Fig. 2 Experimental Configuration

Tables 1 and 2 summarize the RMS values of the estimation errors of the DEKF and the local extended Kalman filter (LEKF) which estimates based only on local measurements without consensus. Table 1 shows that the LEKF is slightly better than the DEKF as for the estimation of the robot positions, though the DEKF enables the robots 1 and 3 to estimate the positions of the other robots which are not directly observed from them. As for the landmark positions, we see from Table 2 that the DEKF performs much better than the LEKF for most cases.

The computation times of the DEKF are about 0.47[ms/step] for the robots 1 and 3, and 0.62 [ms/step] for the robot 2, while the LEKF takes 0.30 [ms/step] for all the robots. The DEKF improves the estimation accuracy at the price of slight increase of computation time.

### 5. CONCLUSION

We have derived the DEKF algorithm for nonlinear systems, and have applied it to the multi-robot SLAM problem. The effectiveness and numerical efficiency of the proposed method have been verified by a numerical experiment with real data.

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